Enroll No. ____________

## Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

**SEMESTER END EXAMINATION NOVEMBER – 2016**

## M.Sc. Mathematics

### 16PMTCC04 - THEORY OF DIFFERENTIAL EQUATIONS

***Duration of Exam – 3 hrs*** ***Semester – I*** ***Max. Marks – 70***

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***Part A (5x2= 10 marks)***

Answer **ALL** questions

1. If $u$ is the complementary function and $v$ is the particular integral for the differential equation $y'' + P(x)y' + Q(x)y = 0$ then prove that the general solution of the differential equation is $y = u + v$.
2. In usual notations prove that(1) $\frac{d}{dx}\left(J_0(x)\right) = -J_1(x)$. (2) $\frac{d}{dx}\left(xJ_1(x)\right) = xJ_0(x)$.
3. Form partial differential equation by eliminating arbitrary constants a and b from the equation $\log(az - 1) = x + ay + b$.
4. Solve the partial differential equation $pq = p + q$.
5. Prove that $(\alpha)_{n+1} = \alpha(\alpha + 1)_n$.

***Part B (5X5 = 25 marks)***

Answer **ALL** questions

**6a.** Solve: $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$, given that $x(0) = 1, y(0) = 0$.

**OR**

**6b.** Find general solution of $y'' + x^2 y = 0$ in terms of power series in $x$.

**7a.** State and prove Rodrigue's formula.

**OR**

**7b.** Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

**8a.** In usual notations, prove that (1) $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$.
(2) $\frac{d}{dx}\left(x^{-n} J_n(x)\right) = x^{-n} J_{n+1}(x)$.

**OR**

**8b.** Prove that the general solution of Lagrange's equation $\mathrm{Pp} + \mathrm{Qq} = \mathrm{R}$ is $\emptyset(\mathrm{u}, \mathrm{v}) = 0$, where $\emptyset$ is an arbitrary function and $\mathrm{u}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = \mathrm{c}_1$ and $\mathrm{v}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = \mathrm{c}_2$ are two independent solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

**9a.** Form partial differential equation by eliminating arbitrary function 'f' from the equation $lx + my + nz = f(x^2 + y^2 + z^2)$

**OR**

**9b.** Find complete integral of partial differential equation $z^2p^2y + 6zpxy + 2zqx^2 + 4x^2y = 0$.

**10a.** Prove that $F(\alpha;\beta;x) = \frac{\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta-\alpha)}\int_0^1 t^{\alpha-1}(1-t)^{\beta-\alpha-1}e^{xt}dt,\ \ \beta > \alpha > 0$.

**OR**

**10b.** Show that the equations $xp - yq = x,\ x^2p + q = xz$ are compatible and find the solution.

***Part C* (5X7 = 35 marks)**

Answer **ALL** questions

**11a.** Using method of variation of parameters, solve the following differential equations.
(1) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. (2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\log x$.

**OR**

**11b.** Solve in series the differential equation $2x(1-x)y'' + (1-x)y' + 3y = 0$ near $x = 0$.

**12a.** Express the following functions in terms of Legendre polynomials.
(1) $f(x) = x^3 + 2x^2 - x - 3$. (2) $g(x) = x^3 - 5x^2 + x + 2$.

**OR**

**12b.** For the Legendre polynomials, prove that $\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0;\ \text{if } m \neq n. \\ \frac{2}{2n+1};\ \text{if } m = n. \end{cases}$ $(n \in \mathbb{N})$.

**13a.** Prove that (1) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$.
(2) $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\cos x + \frac{3}{x}\sin x\right)$.

**OR**

**13b.** Solve Gauss hypergeometric equation: $x(1-x)y'' + [\gamma - (\alpha + \beta + 1)x]y' - \alpha\beta y = 0$.

**14a.** State and derive Charpit's Auxiliary Equations.

**OR**

**14b.** State and derive Jacobi's Auxiliary Equations.

**15a**. Find three successive approximations of the solution of the equation $\frac{dy}{dx} = z, \frac{dz}{dx} = x^3(y+z)$, $y = 1, z = \frac{1}{2}\ when\ x = 0$ using Picard's method.

**OR**

**15b.** Prove that $F(\alpha,\beta;\gamma;x) = \frac{\alpha\beta}{\gamma}F(\alpha+1,\beta+1;\gamma+1;x)$. Hence deduce that
$\frac{d^n}{dx^n}F(\alpha,\beta;\gamma;x) = \frac{(\alpha)_n(\beta)_n}{(\gamma)_n}F(\alpha+n,\beta+n;\gamma+n;x)$.

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